

Electric flux

A clever way to calculate the electric field from a charged conductor is to use Gauss' Law. What we will do is to look at some implications of Gauss' Law. It's also a good time to introduce the concept of *flux*. **Electric flux is a measure of the number of electric field lines passing through an area.** To calculate the flux through a particular surface, multiply the surface area by the component of the electric field perpendicular to the surface. If the electric field is parallel to the surface, no field lines pass through the surface and the flux will be zero. The maximum flux occurs when the field is perpendicular to the surface.

Let's consider a E_n projection of the intensity vector \vec{E} on the perpendicular to an elementary dS area. See fig.2.1:

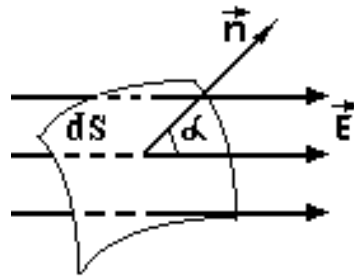


Figure 2.1

The product of E_n by dS is called *the flux of the intensity vector \vec{E}* through this area:

$$d\Phi = E_n dS = E dS \cos \alpha = (\vec{E}, d\vec{S}), \quad (2.1)$$

where $d\vec{S} = dS \times \vec{n}$. A flux of \vec{E} through a closed area is

$$\Phi = \oint_s E_n dS. \quad (2.2)$$

The theorem of Gauss deals with the determination of intensities of various electric fields. This theorem is deduced proceeding from consideration of a shape of charged bodies. In fact it follows from Coulomb's law. Let's consider the electric field near the q as a point charge.

We have got the force, acting on the test charge q_0 :

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{qq_0}{r^2}. \quad (2.3)$$

By definition

$$E = \frac{F}{q_0} \Rightarrow E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}. \quad (2.4)$$

A closed surface is called the surface of Gauss. In general form the theorem of Gauss for electrostatic field in vacuum reads as following: **the flux of the intensity vector in vacuum through an arbitrary closed surface is equal to the sum charges, which are surrounded by this surface, divided by electric constant ϵ_0 :**

$$\Phi = \frac{1}{\epsilon_0} \cdot \sum_i q_i . \quad (2.5)$$

Or more rigorous equation is:

$$\oint_s E_n dS = \frac{1}{\epsilon_0} \cdot \sum_i q_i \quad (2.6)$$

Implications of Gauss' Law

Gauss' Law is a powerful method of electric fields calculating. If you have a solid conducting sphere (e.g., a metal ball) that has a net charge Q on it, you know all the excess charge lies on the outside of the sphere. Gauss' law tells us that the electric field inside the sphere is zero, and the electric field outside the sphere is the same as the field from a point charge with a net charge of Q . That's a pretty neat result.

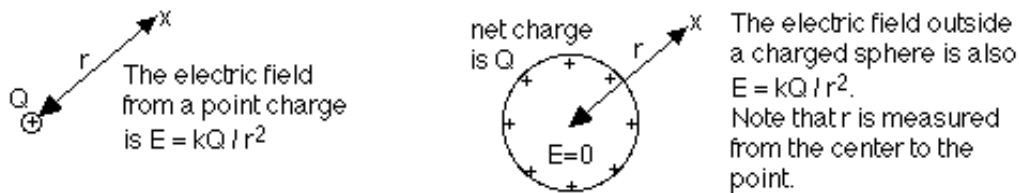
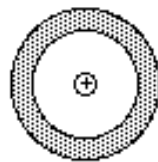


Figure 2.2

The result for the sphere applies whether it's solid or hollow. Let's look at the hollow sphere, and make it more interesting by adding a point charge at the center.



The point charge in the center has a charge $q = +5 \mu\text{C}$

The hollow sphere has a net charge of $Q = -3 \mu\text{C}$

Figure 2.3

What does the electric field look like around this charge inside the hollow sphere? How is the negative charge distributed on the hollow sphere? To find the answers, keep these things in mind:

- The electric field must be zero inside the sphere;
- Outside the sphere, you can find the net electric field by adding, as vectors, the electric field from the point charge alone and from the sphere alone

We know that the electric field from the point charge is given by kq/r^2 . Because the charge is positive, the field points away from the charge. If we took the point charge out of the sphere, the field from the negative charge on the sphere would be zero inside the sphere, and given by kQ/r^2 outside the sphere.

The net electric field with the point charge and the charged sphere, then, is the sum of the fields from the point charge alone and from the sphere alone (except inside the solid part of the sphere, where the field must be zero). This is shown in the figure:

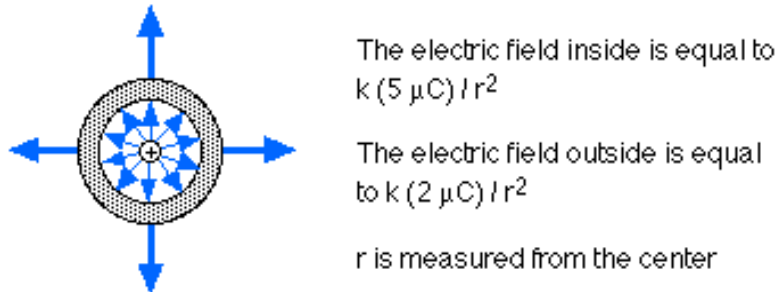


Figure 2.4

How is the charge distributed on the sphere? The electrons must distribute themselves so the field is zero in the solid part. This means there must be -5 microcoulombs of charge on the inner surface, to stop all the field lines from the $+5$ microcoulomb point charge. There must then be $+2$ microcoulombs of charge on the outer surface of the sphere, to give a net charge of $-5 + 2 = -3$ microcoulombs.

1. The electric field intensity, created by a point charge q is a charge, which forms an electric field, r is a distance to a chosen point M . Our aim is to determine the intensity of an electric field at this point. It is reasonable to draw a sphere of Gauss through a given point M . Now we've got:

$\Phi = \frac{q}{\epsilon_0}$, because only the charge q is located inside this sphere. Further

$$\Phi = ES = E \cdot 4\pi r^2 = \frac{q}{\epsilon_0};$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}.$$

In figure 2.5 the value of E is plotted versus r :

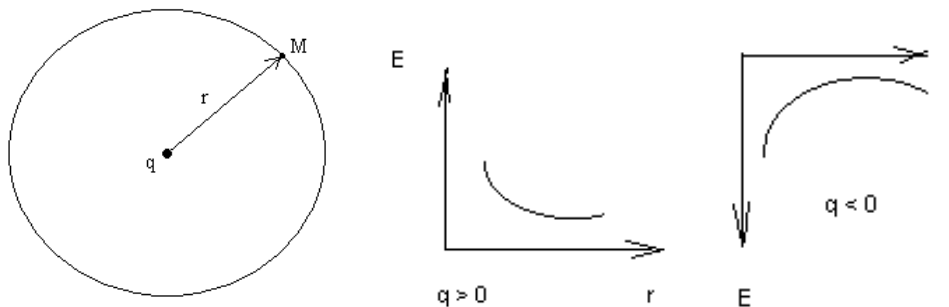
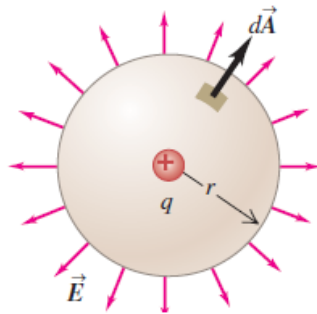


Figure 2.5

Example; A point charge $q = +3.0 \mu\text{C}$ is surrounded by an imaginary sphere of radius $r = 0.2 \text{ m}$ centered on the charge (Fig. 22.9). Find the resulting electric flux through the sphere.



$$\begin{aligned} \Phi_E &= EA = \frac{q}{4\pi\epsilon_0 r^2} 4\pi r^2 = \frac{q}{\epsilon_0} \\ &= \frac{3.0 \times 10^{-6} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} \\ &= 3.4 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C} \end{aligned}$$

22.9 Electric flux through a sphere centered on a point charge.

2. The electric field intensity, created by a hollow sphere. Let us consider that there is a hollow charged sphere of R_0 radius. We should determine its electric field. We mean to find a vector E at any point of space. R_0 is a radius of a real charged sphere; r is the radius of an imagined sphere of Gauss. If $r > R_0$ the theorem of Ostrogradsky is

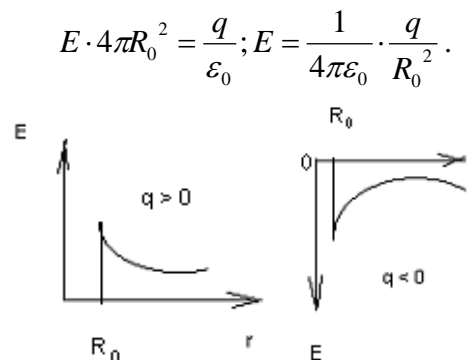


Figure 2.6

If $r < R_0$, $q=0$ inside the sphere and $E=0$.

3. The electric field intensity, created by an infinite charged plane. Let us consider an infinite charged plane. The surface charge density is equal to δ . It is reasonable to use a cylinder as a surface of Gauss. The flux of vector intensity through a side surface is equal to zero. As for butt-end surface it is:

$$2ES = \frac{\sigma S}{\epsilon_0}; q = \sigma S \Rightarrow E = \frac{\sigma}{2\epsilon_0};$$

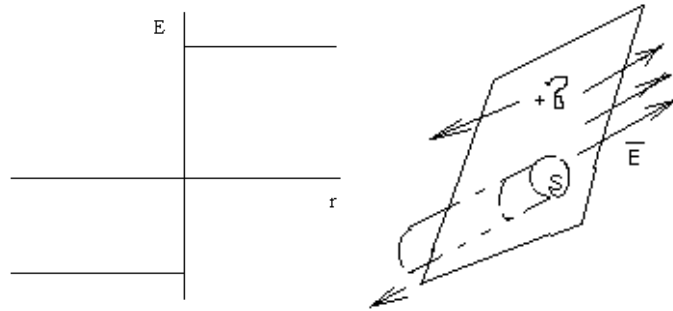


Figure 2.7; Intensity vector E doesn't depend upon r .

4. The electric field of two charged parallel infinite planes. A plane capacitor consists of two planes, which are charged by electricity of the opposite sign (fig.2.6). The surface charge density is $\delta = \frac{dq}{dS}$. The distance between planes is d (fig.2.7).

$$E = E_+ + E_- = 2 \cdot \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}.$$

5. The electric field of an infinite uniformly charged wire. In this case the surface of Gauss is a cylinder. The theorem of Ostrogradsky and Gauss is:

$$E \cdot 2\pi r l = \frac{\lambda l}{\epsilon_0}; q = \lambda l \Rightarrow E = \frac{1}{2\pi\epsilon_0} \cdot \frac{\lambda}{r}.$$

Electric field of various symmetric charge distributions: The following table lists electric fields caused by several symmetric charge distributions. In the table, q , Q , λ , and σ refer to the *magnitudes* of the quantities.

Charge Distribution	Point in Electric Field	Electric Field Magnitude
Single point charge q	Distance r from q	$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$
Charge q on surface of conducting sphere with radius R	Outside sphere, $r > R$	$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$
	Inside sphere, $r < R$	$E = 0$
Infinite wire, charge per unit length λ	Distance r from wire	$E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}$
Infinite conducting cylinder with radius R , charge per unit length λ	Outside cylinder, $r > R$	$E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}$
	Inside cylinder, $r < R$	$E = 0$

Solid insulating sphere with radius R , charge Q distributed uniformly throughout volume	Outside sphere, $r > R$	$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$
	Inside sphere, $r < R$	$E = \frac{1}{4\pi\epsilon_0} \frac{Qr}{R^3}$
Infinite sheet of charge with uniform charge per unit area σ	Any point	$E = \frac{\sigma}{2\epsilon_0}$
Two oppositely charged conducting plates with surface charge densities $+\sigma$ and $-\sigma$	Any point between plates	$E = \frac{\sigma}{\epsilon_0}$
Charged conductor	Just outside the conductor	$E = \frac{\sigma}{\epsilon_0}$

Work by electrostatic forces. Electric potential

Let us move a test positive unit charge q_0 from the first point to the second one along line L and calculate the work.

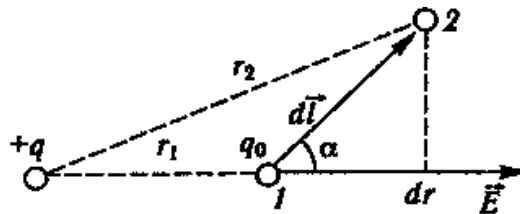


Figure 2.8

The elementary part of length dl is so small, that the electric field intensity \vec{E} may be taken as a constant in magnitude and direction within the limits of dl . The total work, done in this small interval is

$$dA = F dl \cos \alpha = \frac{1}{4\pi\epsilon_0} \cdot \frac{qq_0}{r^2} dl \cos \alpha, \quad (2.7)$$

as $dl \cos \alpha = dr$,

$$dA = \frac{1}{4\pi\epsilon_0} \cdot \frac{qq_0}{r^2} dr. \quad (2.8)$$